Background-Independent Gravitational Waves.

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(Dated: February 7, 2008)

A Hamiltonian linearization of the rest-frame instant form of tetrad gravity, where the Hamiltonian is the weak ADM energy E_{ADM}, in a completely fixed (non harmonic) 3-orthogonal Hamiltonian gauge is defined. For the first time this allows to find an explicit solution of all the Hamiltonian constraints and an associated linearized solution of Einstein's equations. It corresponds to background-independent gravitational waves in a well defined post-Minkowskian Christodoulou-Klainermann space-time.

ADM canonical gravity has always been important for the Cauchy problem and for the interpretation of general relativity, but till now it has not produced new exact solutions of Einstein's equations due to the difficulty in solving its first class constraints. Instead recently there have been the first attempts to use it for numerical simulations of stars and of the coalescence of binary systems of neutron stars and/or black holes. In this letter we will show that it is possible to find an approximate solution of all the constraints and then of vacuum Einstein equations by defining a Hamiltonian linearization of ADM tetrad gravity in a completely fixed Hamiltonian gauge, corresponding to a system of non-harmonic 4-coordinates. This approximate solution corresponds to a *background-independent gravitational wave* and defines a post-Minkowskian space-time, linearization of a Christodoulou - Klainermann space-time [1]. Here only the general method and the main results will be stated, since all the detailed calculations are contained in Ref.[2].

As shown in Ref.[3], it is possible to define a *rest-frame instant form* of ADM metric gravity, without or with matter, for globally hyperbolic, topologically trivial, asymptotically flat at spatial infinity space-times, in such a way that in the limit of vanishing Newton constant the rest-frame instant form of parametrized Minkowski theories [4] is recovered. This is possible if the allowed 3+1 splittings of the space-time are associated with foliations whose leaves are space-like hyper-surfaces $\Sigma_\tau^{(WSW)}$ [named Wigner-Sen-Witten (WSW) surfaces [2]; τ is a mathematical time labeling the leaves; $\vec{\sigma}$ are curvilinear coordinates on Σ_τ] tending, in a direction-independent way, to Minkowski hyper-planes orthogonal to the ADM 4-momentum at spatial infinity [5]. The *SPI group* of asymptotic symmetries at spatial infinity is reduced to the Poincarè group with the strong ADM Poincarè charges as generators. The 4-metric is assumed to have a well defined direction-independent limit at spatial infinity. It can be shown [3] that in the rest-frame instant form of gravity the Dirac Hamiltonian, after the addition of a suitable surface term and a suitable splitting of the lapse and shift functions in asymptotic and bulk parts, is weakly equal to the *weak ADM energy* (see also Ref.[6]), which is an integral over 3-space, weakly equal to the strong ADM energy (surface form). In Ref.[7], after the introduction of a new parametrization of tetrad gravity [8], its rest-frame instant form is introduced. In this form of dynamics we have the extra constraints $\vec{P}_{ADM} \approx 0$, defining the *rest frame of the universe* like in Christodoulou - Klainermann space-times.

The canonical variables of this re-formulation of tetrad gravity are 3 boost parameters $\varphi_{(a)}(\tau,\vec{\sigma})$, 3 angles $\alpha_{(a)}(\tau,\vec{\sigma})$, the bulk parts $n(\tau,\vec{\sigma})$, $n_r(\tau,\vec{\sigma})$ of the lapse and shift functions, cotriads ${}^3e_{(a)r}(\tau,\vec{\sigma})$ over $\Sigma_\tau^{(WSW)}$ and the 16 conjugate momenta. There are 14 first class constraints: 7 are given by the vanishing of the boost, lapse and shift momenta; 6 are the generators of rotations and passive 3-diffeomorphisms and the last one is the super-hamiltonian constraint. This last constraint is the generator of the Hamiltonian gauge transformations which modify an allowed 3+1 splitting of space-time into another allowed one: as a consequence the physical results are independent from the choice of the 3+1 splitting. It can be shown [3] that the super-hamiltonian constraint is an equation for the determination of the conformal factor $\phi(\tau,\vec{\sigma}) = det(\,{}^3g(\tau,\vec{\sigma})\,)^{1/12}$ of the 3-metric on Σ_τ, namely it has to be interpreted as the Lichnerowicz

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equation.

By means of a point Shanmugadhasan canonical transformation [9] 13 constraints (with the exception of the super-hamiltonian one) become momenta of the new canonical basis (Abelianization of the first class constraints). The conjugate variables are 13 Abelianized gauge variables (describing generalized inertial effects). The canonical basis is completed with i) the conformal factor $\phi(\tau,\vec{\sigma})$ (to be determined by the Lichnerowicz equation) and $\pi_\phi(\tau,\vec{\sigma})$; ii) two (non-tensorial) conjugate pairs $r_{\bar a}(\tau,\vec{\sigma})$, $\pi_{\bar a}(\tau,\vec{\sigma})$, $\bar a = 1,2$, of Dirac observables (DO; the deterministically predictable degrees of freedom of the gravitational field, describing generalized tidal effects). See Refs.[3, 7, 10] for the interpretational problems.

By adding 14 suitable gauge fixing constraints (one of them must be $\pi_\phi(\tau,\vec{\sigma}) \approx 0$), we get a completely fixed 3-orthogonal Hamiltonian gauge in which the 3-metric is diagonal. It corresponds, on the solutions of Einstein's equations, to a unique non-harmonic 4-coordinate system [2]. In this radiation gauge all 4-tensors are expressed only in terms of the two pairs of conjugate DO $r_{\bar a}(\tau,\vec{\sigma})$, $\pi_{\bar a}(\tau,\vec{\sigma})$. In particular we get ${}^3g_{rs} = \delta_{rs}\,\phi^4\, exp(\frac{2}{\sqrt{3}}\sum_{\bar a}\gamma_{\bar a r}\, r_{\bar a})$, where the $\gamma_{\bar a r}$'s are suitable constants. The DO $r_{\bar a}(\tau,\vec{\sigma})$ *replace the two polarizations of the harmonic TT gauge*. Let us remark that in absence of a background we do not have a theory of a spin 2 wave propagating over Minkowski space-time but a genuine Einstein space-time.

It can be shown [2] that in the new canonical basis the solution of the 6 rotation and super-momentum constraints reduces to solve a set of quasi-linear partial differential equations of elliptic type. Instead the Lichnerowicz equation becomes a in-tractable integro-differential equation for $\phi(\tau,\vec{\sigma})$. Also the bulk lapse and shift functions are determined by integral equations.

Then, without never introducing a background 4-metric, we make a *weak field approximation* and a Hamiltonian linearization by systematically discarding terms of order $O(r^2_{\bar a})$ in the Lichnerowicz equation and $O(r^3_{\bar a})$ in the weak ADM energy.

A) We assume $|r_{\bar a}(\tau,\vec{\sigma})| << 1$ on each WSW hypersurface and $|\partial_u r_{\bar a}(\tau,\vec{\sigma})| \sim \frac{1}{L}O(r_{\bar a})$, $|\partial_u\partial_v r_{\bar a}(\tau,\vec{\sigma})| \sim \frac{1}{L^2}O(r_{\bar a})$, where L is a *big enough characteristic length interpretable as the reduced wavelength $\lambda/2\pi$ of the resulting gravitational waves*. Since the conjugate momenta $\pi_{\bar a}(\tau,\vec{\sigma})$ have the dimensions of $\frac{action}{L^3}$, i.e. of $\frac{k}{L}$ with $k = \frac{c^3}{16\pi G}$, we assume $|\pi_{\bar a}(\tau,\vec{\sigma})| \sim \frac{k}{L}O(r_{\bar a})$, $|\partial_u\pi_{\bar a}(\tau,\vec{\sigma})| \sim \frac{k}{L^2}O(r_{\bar a})$, $|\partial_u\partial_v\pi_{\bar a}(\tau,\vec{\sigma})| \sim \frac{k}{L^3}O(r_{\bar a})$. Therefore, $r_{\bar a}(\tau,\vec{\sigma})$ and $\pi_{\bar a}(\tau,\vec{\sigma})$ are *slowly varying over the length* L (for $r_{\bar a}, \pi_{\bar a} \rightarrow 0$ we get the void space-times of Ref.[3]). It can be shown [2] that the Riemann tensor of our space-time is of order $\frac{1}{L\,k}\,O(\pi_{\bar a}) = \frac{1}{L^2}\,O(r_{\bar a}) \approx \mathcal{R}^{-2}$, where $\mathcal{R}$ is the mean radius of curvature. Therefore the *requirements of the weak field approximation are satisfied*: i) $\mathcal{A} = O(r_{\bar a})$, if $\mathcal{A}$ is the amplitude of the gravitational wave; ii) $\frac{L}{\mathcal{R}} = O(r_{\bar a})$, namely $\frac{\lambda}{2\pi} << \mathcal{R}$.

B) We also assume $q(\tau,\vec{\sigma}) = 2\,ln\,\phi(\tau,\vec{\sigma}) \sim O(r_{\bar a})$, $\partial_u q(\tau,\vec{\sigma}) \sim \frac{1}{L}O(r_{\bar a})$, $\partial_u\partial_v q(\tau,\vec{\sigma}) \sim \frac{1}{L^2}O(r_{\bar a})$, so that we get $\phi(\tau,\vec{\sigma}) = e^{q(\tau,\vec{\sigma})/2} \approx 1 + \frac{1}{2}q(\tau,\vec{\sigma}) + O(r^2_{\bar a})$ for the conformal factor. The Lichnerowicz equation becomes the linear partial differential equation $\triangle q(\tau,\vec{\sigma}) = \frac{1}{2\sqrt{3}}\sum_{u\bar a}\gamma_{\bar a u}\partial^2_u r_{\bar a}(\tau,\vec{\sigma}) + \frac{1}{L^2}O(r^2_{\bar a})$ for $q(\tau,\vec{\sigma})$, whose solution vanishing at spatial infinity is

$$q(\tau,\vec{\sigma}) = \frac{-1}{2\sqrt{3}}\sum_{u\bar a}\gamma_{\bar a u}\int d^3\sigma_1\,\frac{\partial^2_{1u} r_{\bar a}(\tau,\vec{\sigma}_1)}{4\pi|\vec{\sigma}-\vec{\sigma}_1|} + O(r^2_{\bar a}). \quad (1)$$

Then we are able to solve the quasi-linear partial differential equations equivalent to the 6 rotation and super-momentum constraints. Their solution implies that the cotriad momenta have the following expression in terms of DO's

$$\begin{aligned}{}^3\pi^r_{(a)}(\tau,\vec{\sigma}) &= \sqrt{3}\sum_{\bar a}\gamma_{\bar a r}\delta^r_{(a)}\pi_{\bar a}(\tau,\vec{\sigma}) + \quad (2)\\ &+\frac{\sqrt{3}}{2}[1-\delta^r_{(a)}]\sum_{\bar a u}\gamma_{\bar a u}[1-2(\delta_{ru}+\delta_{au})]\,\cdot\\ &\cdot\,\frac{\partial^2}{(\partial\sigma^u)^2}\int_{\sigma^r}^{\infty} d\sigma^r_1\int_{\sigma^a}^{\infty} d\sigma^a_1\,\pi_{\bar a}(\tau,\sigma^r_1\sigma^a_1\sigma^{k\neq r,a}) + O(r^2_{\bar a}),\end{aligned}$$

with the restriction $\int_{-\infty}^{+\infty} d\sigma^r\,\pi_{\bar a}(\tau,\vec{\sigma}) = 0$, $r = 1,2,3$. Hamilton equations are compatible with these restrictions if we also have $\int_{-\infty}^{+\infty} d\sigma^r\, r_{\bar a}(\tau,\vec{\sigma}) = 0$.

The integral equations for the lapse and shift functions yield the following solutions [the signature of the 4-metric is $\epsilon(+---)$ with $\epsilon = \pm 1$] $N(\tau,\vec{\sigma}) = -\epsilon + n(\tau,\vec{\sigma}) = -\epsilon + O(r^2_{\bar a})$, $N_r(\tau,\vec{\sigma}) = n_r(\tau,\vec{\sigma}) = -\epsilon\,{}^4g_{\tau r}(\tau,\vec{\sigma})$ and

$$\begin{aligned}n_r(\tau,\vec{\sigma}) \;=\; &\frac{\partial}{\partial\sigma^r}\Big(\frac{2\sqrt{3}\pi G}{c^3}\sum_{\bar a v}\gamma_{\bar a v}\Big[\sum_{ua, u\neq a}[1-2(\delta_{uv}+\delta_{av})]\int_{-\infty}^{\sigma^u}d\sigma^u_1\int_{-\infty}^{\sigma^a}d\sigma^a_1\int_{\sigma^u_1}^{\infty}d\sigma^u_2\int_{\sigma^a_1}^{\infty}d\sigma^a_2\,\frac{\partial^2\pi_{\bar a}(\tau,\sigma^u_2\sigma^a_2\sigma_2^{k\neq u,a})}{(\partial\sigma^v_2)^2}|_{\sigma^k_2=\sigma^k} -\\ &-\;2\sum_{u\neq r}[1-2(\delta_{uv}+\delta_{rv})]\int_{-\infty}^{\sigma^r}d\sigma^r_1\int_{-\infty}^{\sigma^u}d\sigma^u_1\int_{\sigma^r_1}^{\infty}d\sigma^r_2\int_{\sigma^u_1}^{\infty}d\sigma^u_2\,\frac{\partial^2\pi_{\bar a}(\tau,\sigma^r_2\sigma^u_2\sigma_2^{k\neq r,u})}{(\partial\sigma^v_2)^2}|_{\sigma^k_2=\sigma^k}\Big]\Big) + O(r^2_{\bar a}). \quad (3)\end{aligned}$$

After the solution of all the constraints (super-hamiltonian one included), the *4-metric* of our linearized

space-time, in $\Sigma_\tau^{(WSW)}$-adapted coordinates, in our 3-orthogonal gauge, becomes (the form of a perturbation of the Minkowski metric in Cartesian coordinates is used *only to visualize* the deviations from special relativity) ${}^4g_{AB}(\tau,\vec{\sigma}) = {}^4\eta_{AB} + {}^4h_{AB}(\tau,\vec{\sigma})$. We have ${}^4h_{\tau\tau}(\tau,\vec{\sigma}) = 0 + O(r_{\bar a}^2)$, ${}^4h_{\tau r}(\tau,\vec{\sigma}) = -\epsilon n_r(\tau,\vec{\sigma})$ and

$$ {}^4h_{rs}(\tau,\vec{\sigma}) \;=\; -\frac{2\epsilon}{\sqrt{3}}\sum_{\bar a}\Big[\gamma_{\bar a r} r_{\bar a}(\tau,\vec{\sigma}) - \frac{1}{2}\sum_u \gamma_{\bar a u}\int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar a}(\tau,\vec{\sigma}_1)}{4\pi|\vec{\sigma}-\vec{\sigma}_1|}\Big]\delta_{rs} + O(r_{\bar a}^2). \tag{4} $$

As said the Hamiltonian is the weak ADM energy. After the solution of all the constraints its quadratic part in the DO's is [see Eq.(2.15) of Ref.[2] for the xpression of the kernel $\mathcal{T}^{(o)u}_{(a)r}(\vec{\sigma},\vec{\sigma}_1)$]

$$ \begin{aligned} E_{ADM} \;=\; & \frac{12\pi G}{c^3}\int d^3\sigma \sum_{\bar a}\Big[\pi^2_{\bar a}(\tau,\vec{\sigma}) + \sum_{\bar a\bar b}\sum_{rs}\gamma_{\bar a r}\gamma_{\bar b s}\int d^3\sigma_1 d^3\sigma_2 \sum_u \mathcal{T}^{(o)u}_{(a)r}(\vec{\sigma},\vec{\sigma}_1)\mathcal{T}^{(o)u}_{(a)s}(\vec{\sigma},\vec{\sigma}_2)\,\pi_{\bar a}(\tau,\vec{\sigma}_1)\,\pi_{\bar b}(\tau,\vec{\sigma}_2)\Big] - \\ -\; & \frac{c^3}{16\pi G}\sum_r\int d^3\sigma\Big[\frac{1}{6}\Big(\sum_{\bar a u}\gamma_{\bar a u}\frac{\partial}{\partial\sigma^r}\int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar a}(\tau,\vec{\sigma}_1)}{4\pi|\vec{\sigma}-\vec{\sigma}_1|}\Big)^2 - \frac{1}{3}\sum_{\bar a}\Big(\partial_r r_{\bar a}(\tau,\vec{\sigma})\Big)^2 + \frac{2}{3}\Big(\sum_{\bar a}\gamma_{\bar a r}\partial_r r_{\bar a}(\tau,\vec{\sigma})\Big)^2 - \\ -\; & \frac{1}{3}\sum_{\bar a\bar b}\sum_u \gamma_{\bar a r}\partial_r r_{\bar a}(\tau,\vec{\sigma})\gamma_{\bar b u}\frac{\partial}{\partial\sigma^r}\int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar b}(\tau,\vec{\sigma}_1)}{4\pi|\vec{\sigma}-\vec{\sigma}_1|}\Big] + O(r^3_{\bar a}). \end{aligned} \tag{5} $$

The rest-frame condition, i.e. the vanishing of the weak ADM 3-momentum, is $P^r_{ADM} = -\int d^3\sigma\, \sum_{\bar c}\, \pi_{\bar c}(\tau,\vec{\sigma})\,\partial_r\, r_{\bar c}(\tau,\vec{\sigma}) \approx 0$. Since we are in an instant form of the dynamics both the ADM 3-momentum and angular momentum ($J^{rs}_{ADM} = \int d^3\sigma\, \sum_{\bar c}\, \pi_{\bar c}(\tau,\vec{\sigma})\,(\sigma^r\,\partial_s - \sigma^s\,\partial_r)\, r_{\bar c}(\tau,\vec{\sigma})$) have the same form as in a free theory. Instead the ADM energy and boosts have a complicated form. Notwithstanding this fact, the study of the Hamilton equations for the DO's imply [2] that, even if we are not in a harmonic gauge, the $r_{\bar a}(\tau,\vec{\sigma})$ satisfy the wave equation

$$ \Box\, r_{\bar a}(\tau,\vec{\sigma}) \,\circeq\, 0. \tag{6} $$

A *class of solutions* of the Hamilton equations vanishing correctly at spatial infinity, satisfing the rest frame condition $P^r_{ADM} = 0$ and every restriction, is

$$ \begin{aligned} r_{\bar a}(\tau,\vec{\sigma}) &= \\ &= C_{\bar a}\int \frac{d^3k}{(2\pi)^3}\,\frac{(k_1k_2k_3)^2 e^{-\vec{k}^2}}{|\vec{k}|}\Big[e^{-i(|\vec{k}|\tau-\vec{k}\cdot\vec{\sigma})} + e^{i(|\vec{k}|\tau-\vec{k}\cdot\vec{\sigma})}\Big] \\ &= -\frac{4\,C_{\bar a}}{(2\pi)^2}\,\frac{\partial^6}{\partial^2\sigma_1\partial^2\sigma_2\partial^2\sigma_3}\,\frac{(1+|\vec{\sigma}|^2-\tau^2)}{[1+(|\vec{\sigma}|+\tau)^2]\,[1+(|\vec{\sigma}|-\tau)^2]}, \end{aligned} \tag{7} $$

$$ \begin{aligned} \pi_{\bar a}(\tau,\vec{\sigma}) &= -i\int \frac{d^3k}{(2\pi)^3}\,\frac{(k_1k_2k_3)^2}{|\vec{k}|}\,e^{-\vec{k}^2}\,|\vec{k}| \\ &\sum_{\bar b} A^{-1}_{\bar a\bar b}(\vec{k})\,C_{\bar b}\,\Big[e^{-i(|\vec{k}|\,\tau-\vec{k}\cdot\vec{\sigma})} - e^{i(|\vec{k}|\,\tau-\vec{k}\cdot\vec{\sigma})}\Big]. \end{aligned} \tag{8} $$

It describes standing waves and the two constants $C_{\bar 1}$, $C_{\bar 2}$ have to be expressed in terms of the two boundary constants $M = E_{ADM}$, $S = |\vec{J}_{ADM}|$, defining the mass and spin of the post-Minkowskian space-time. *In absence of matter, the rest-frame condition destroys the transversality property of the TT harmonic gauge plane waves.*

In Ref.[2], after a study of both the geodesic equation and the geodesic deviation equation in our gauge, we solve the latter equation numerically for a sphere of test particles at rest around the origin of the 3-coordinates on a WSW hyper-surface for the previous solution. We obtain the two 3-dimensional deformation patterns replacing the usual 2-dimensional ones for the polarization in the TT harmonic gauge: i) in figure 1 there is the deformation pattern for the case $C_{\bar 1} \neq 0$, $C_{\bar 2} = 0$, namely for $r_{\bar 1}(\tau,\vec{\sigma}) \neq 0$, $r_{\bar 2}(\tau,\vec{\sigma}) = 0$; ii) in figure 2 that for the case $C_{\bar 1} = 0$, $C_{\bar 2} \neq 0$, namely for $r_{\bar 1}(\tau,\vec{\sigma}) = 0$, $r_{\bar 2}(\tau,\vec{\sigma}) \neq 0$. In the two figures are reported the snapshots at three different times ($t = -1, -0.5, 0$) of the sphere of particles originally at rest (bottom) and the time evolution (from $t = -3$ to $t = 3$) of the six particles at the intersection of the three axes and the sphere of particle (top), whose initial 3-coordinates are $(1,0,0)$ and $(-1,0,0)$ on the x-axis, $(0,1,0)$ and $(0,-1,0)$ on the y-axis, $(0,0,1)$ and $(0,0,-1)$ on the z-axis, respectively. Only the i coordinates are reported for the two particles on the i axis, since they remain on it.

Let us remark that till now we have a treatment of the generation of gravitational waves from a compact localized source of size R and mean internal velocity v only [11] for *nearly Newtonian slow motion* sources for which $v << c$, $\frac{\lambda}{2\pi} >> R$: outgoing gravitational waves are observed in the *radiation zone* (far field, $r >> \frac{\lambda}{2\pi}$), while deep in the *near zone* ($R < r << \frac{\lambda}{2\pi}$), for example $r \leq 1000\,\frac{\lambda}{2\pi}$, vacuum Newtonian gravitation theory is valid. On the contrary with our approach in suit-

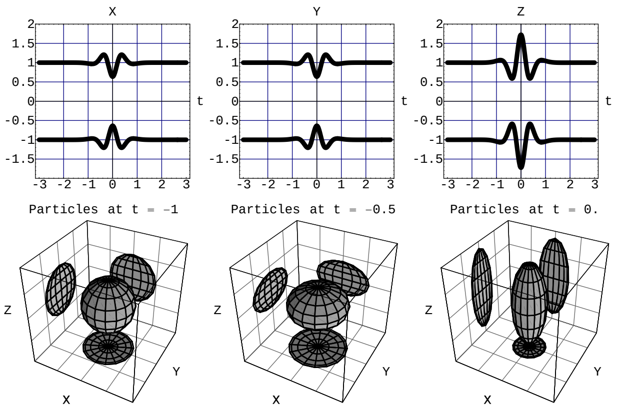

FIG. 1: Deformation of a sphere of particle at rest induced by the passage of the gravitational wave packet for $C_{\bar{1}} \neq 0$.

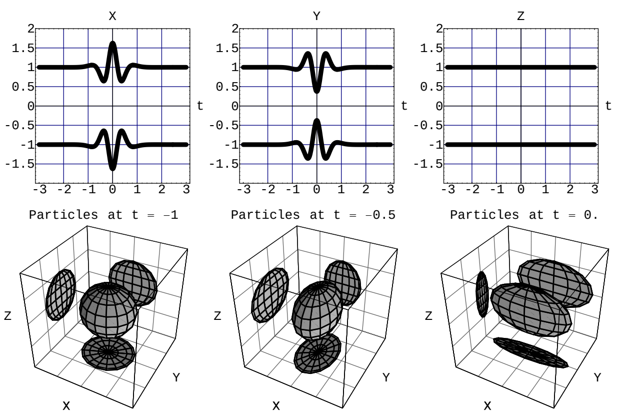

FIG. 2: Deformation of a sphere of particle at rest induced by the passage of the gravitational wave packet for $C_{\bar{2}} \neq 0$.

able 4-coordinates we are going to obtain a *weak field approximation but with fast relativistic motion in the source* subject to the restriction that the total invariant mass and the mass currents are compatible with the weak field requirement. This is enough to get relativistic results conceptually equivalent to the re-summation of the post-Newtonian expansion. Therefore the results about background-independent gravitational waves in post-Minkowskian space-times *open the possibility, after the introduction of matter, to study the emission of gravitational waves from relativistic sources without any kind of post-Newtonian approximation.* For instance this is the case for the relativistic motion (but still in the weak field regime) of the binaries before the beginning of the final inspiral phase: it is known that in this phase the post-Newtonian approximation does not work and that, till now, only numerical gravity may help. In a future paper we will add a relativistic perfect fluid, described by suitable Lagrangian [12] or Eulerian [13] variables, to tetrad gravity, we will define a Hamiltonian linearization of the system in our completely fixed 3-orthogonal gauge, we will find the Hamilton equations for the DO's both of the gravitational field and of the fluid, we will find the relativistic version of the Newton and gravitomagnetic action-at-a-distance potentials and of the generalized tidal effects acting inside the fluid and finally, by using a multipolar expansion, we will find the relativistic counterpart of the post-Newtonian quadrupole emission formula.

Moreover we will have to explore whether our Hamiltonian approach is suitable for doing Hamiltonian numerical gravity on the full non-linearized theory.

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